Math 240 **FINAL EXAM** **December 17, 2010**

Circle one: **Professor Shatz**
Professor Wylie
Professor Ziller

Name: ______________________________

Penn Id#: ______________________________

Signature: ______________________________

TA: ______________________________

Recitation Day and Time: ______________________

Please show your work clearly. A correct answer with no work is worth 0 points. Circle your answer. Continue on the back of the page if necessary (with an arrow on the front page pointing to it). If you do work in a blue book, number the page with the problem number, put your name and data on the blue book cover, and start a **new** page for each separate problem. Each problem is worth 10 points. You are NOT allowed to use a calculator. Your double sided 8.5″ by 11″ sheet of notes must be hand written in your own hand writing (no copies allowed). No books, tables, notes, calculators, computers, cell phones or any other electronic equipment is allowed.

(Do not fill these in; they are for grading purposes only.)

1) ________	6) ________	11) ________
2) ________	7) ________	12) ________
3) ________	8) ________	13) ________
4) ________	9) ________	14) ________
5) ________	10) ________	15) ________

Total ______________

1. Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$$

One of the two combinations, AA^TB or ABA^T, is well defined. What is the determinant of the well defined matrix?

2. For which values of k are the vectors

$$\begin{pmatrix} 0 \\ 1 \\ k \end{pmatrix}, \begin{pmatrix} 1 \\ k \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix}$$

linearly dependent?

3. Let

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}.$$

Suppose that $\det(A) = 2$ and let

$$B = \begin{pmatrix} b_1 & a_1 & 2c_1 - a_1 \\ b_2 & a_2 & 2c_2 - a_2 \\ b_3 & a_3 & 2c_3 - a_3 \end{pmatrix}.$$

What is $\det(2AB)$

4. For what values of k is the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & k \\ 0 & 0 & k \end{pmatrix}$$

diagonalizable?

5. Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & -1 & -1 \end{pmatrix}$$

and compute $\det A$.

6. Find the solution to the system of differential equations

$$X' = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix} X$$

where $X(t) = (x(t), y(t))$ with initial conditions $X(0) = (1, 2)$.

7. For the differential equation

$$x^2y'' - 6y = 0$$

we seek the solution which is bounded at $x = 0$ and equals 2 when $x = 1$. Write down the general solution first and then determine the bounded solution satisfying the above conditions.

8. Find the solution of the differential equation

$$y'' + y = e^x$$

with initial conditions $y(0) = 0$ and $y'(0) = \frac{3}{2}$.

9. Find the solution of the differential equation

$$y'' - 4y = e^{2x}$$

with initial conditions $y(0) = 0$ and $y'(0) = 0$.

10. The differential equation

$$xy'' - 2y = 0$$

has a power series solution of the form:

$$y = x(a_0 + a_1x + a_2x^2 + a_3x^3 \dots)$$

We seek the solution for which $y'(0) = 1$. Determine the recursion relation for the coefficients and write down the first 4 terms.

11. Determine the value of the line integral

$$\int_C 6x^2 e^{2x^3-2y^3}\,dx - 6y^2 e^{2x^3-2y^3}\,dy$$

where C is the semicircle $x^2+y^2=1,\ y \geq 0$, traversed from $(-1,0)$ to $(1,0)$.

12. Find the value of a so that the line integral

$$\int_C ay^3zdx + xy^2zdy + \frac{1}{3}xy^3dz$$

is independent of the path, C, taken between any two given points.

13. Find the outward flux

$$\iint_S \mathbf{F} \cdot \mathbf{n}\ \mathbf{dS}$$

of the vector field

$$\mathbf{F} = 4xy^2\mathbf{i} + 3y\mathbf{j} + 4zx^2\mathbf{k}$$

where the surface S is the boundary of the region $1 \le x^2 + y^2 \le 4,\ 0 \le z \le 1.$

14. Compute

$$\iint_S \mathbf{curlF} \cdot \mathbf{n}\ \mathbf{dS}$$

where

$$\mathbf{F} = xz\mathbf{i} + (zy - 2y)\mathbf{j} + y^2z\mathbf{k}$$

and S is the cone $z^2 = x^2 + y^2$ with $0 \le z \le 2$ and $\mathbf{n}$ the outward (i.e. downward pointing) normal.

15. Consider the curve C, traced counter clockwise, which is the ellipse $\frac{1}{4}x^2 + \frac{1}{9}y^2 = 1$. (C lies between a circle of radius 1 and a circle of radius 5.)

Compute the line integral

$$\int_C \mathbf{F} \cdot \mathbf{dr}$$

where

$$\mathbf{F(x, y)} = \frac{-y}{x^2 + y^2}\mathbf{i} + \frac{x}{x^2 + y^2}\mathbf{j}$$